HURLSTONE AGRICULTURAL HIGH SCHOOL

STUDENT'S NAME: _____

TEACHER'S NAME: _____

2023

HSC ASSESSMENT TASK 4
TRIAL EXAMINATION

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using a black or blue pen
- NESA approved calculators may be used
- A reference sheet and a Normal Distribution Table has been provided in the Section I booklet
- For questions in Section II, **show all relevant mathematical reasoning and/or calculations**

Total marks: 100

Section I: 10 marks (pages 2 – 4)

- Attempt Questions 1 – 10.
  A multiple-choice answer sheet has been provided
- Allow about 15 minutes for this section

Sections II – VII: 90 marks (pages 11 -57)

- Attempt Questions 11 – 41.
  Write your solutions in the spaces provided
- There are **6 separate question/answer booklets**
  Extra working pages are available if required
- Allow about 2 hours and 45 minutes for these sections

This examination paper is not to be removed from the Examination Centre

SECTION I

10 marks
Attempt Questions 1 - 10
Allow about 15 minutes on this section.

Use the multiple-choice answer sheet provided for Questions 1 – 10.

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1. The linear function $f(x) = 5 - x$ has range $[-4, 5)$.
   Which of the following gives the domain of the function?

   A. $(0, 9]$

   B. $(0, 1]$

   C. $[5, -4)$

   D. $(-9, 0)$

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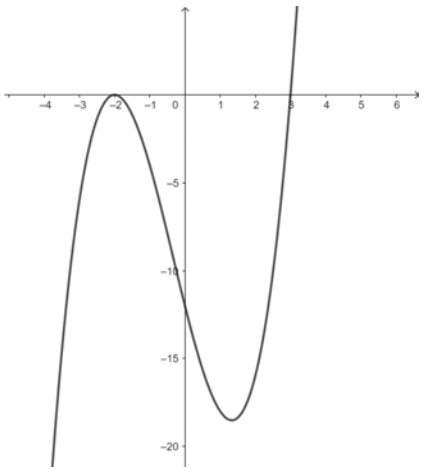
2. Which of the following is the graph of $P(x) = (3 - x)(x + 2)^2$?

A.



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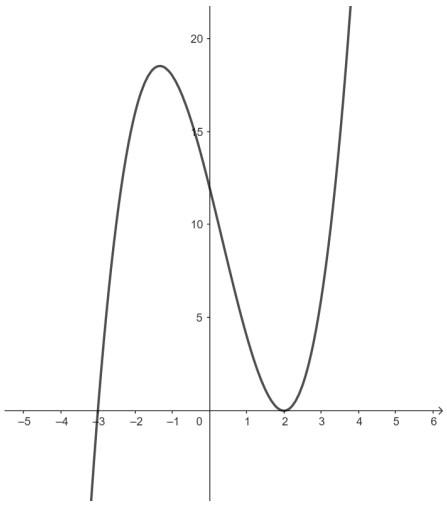
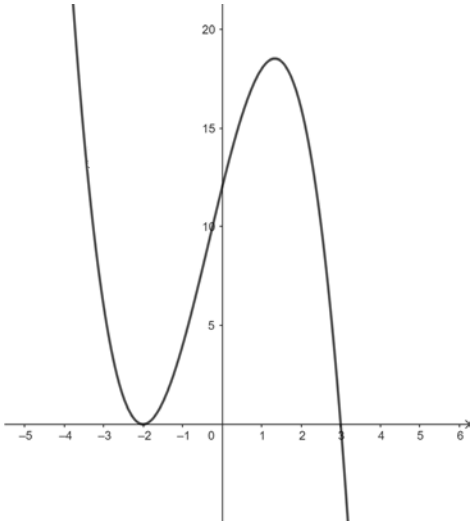
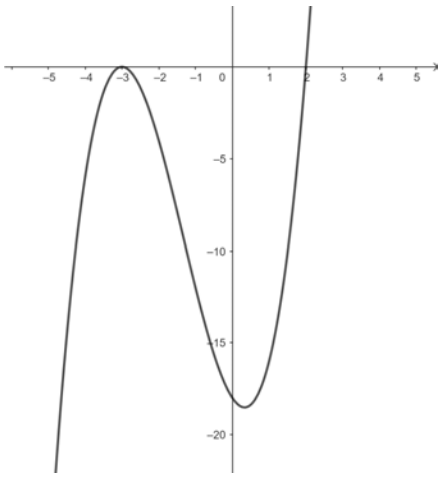


C.



D.



Section I continued on next page ...

3. Which of the following is the solution to $\int \sin\left(\frac{x}{5}\right) dx$?

A. $-\frac{1}{5}\cos\left(\frac{x}{5}\right) + c$

B. $-5\cos 5x + c$

C. $-5\cos\left(\frac{x}{5}\right) + c$

D. $5\cos\left(\frac{x}{5}\right) + c$

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4. What is the maximum area of a rectangle with perimeter 620 mm?

A. 24 025 mm^2

B. 227025 mm^2

C. 24 000 mm^2

D. 24 075 mm^2

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5. The displacement, s metres, at time t seconds of an object moving in a straight line is given by

$$s = t^3 - 6t^2 - 8t - 5\,.$$

What is the equation of its acceleration, a, in terms of t?

A. $a = t^3 - 6t^2 - 8t - 5.$

B. $a = 3t^3 - 6t^2 - 8t - 5.$

C. $a = 6t - 12.$

D. $a = 6.$

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6. What is the derivative with respect to x of

$$\frac{1}{(2x+5)^3}\ ?$$

A. $\frac{6x}{3(2x+5)^2}$

B. $\frac{-6}{(2x+5)^4}$

C. $\frac{-3}{(2x+5)^4}$

D. $\frac{-6x}{(2x+5)^4}$

Section I continued on next page ...

7. Over the domain $a \leq x \leq b$, a function is always increasing and is always concave down. Which pair of statements is correct?

A. $f'(x) < 0, f''(x) < 0$

B. $f'(x) < 0, f''(x) > 0$

C. $f'(x) > 0, f''(x) > 0$

D. $f'(x) > 0, f''(x) < 0$

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8. A data set has a mean of 50 and a standard deviation of 7.
A value of 40 is added to the data set.

How does the new value affect the mean and standard deviation of the data set?

A. The mean increases and the standard deviation increases

B. The mean increases and the standard deviation decreases

C. The mean decreases and the standard deviation increases

D. The mean decreases and the standard deviation decreases

---

9. One cycle of a cosine curve has a maximum value at $\left(\frac{\pi}{4}, 5\right)$ and a minimum value at $\left(\frac{3\pi}{4}, -5\right)$.
Which of the following is the equation of this curve?

A. $y = 5\cos\left(x - \frac{\pi}{4}\right)$

B. $y = 5\cos\left(2x - \frac{\pi}{4}\right)$

C. $y = 5\cos\left(x - \frac{\pi}{2}\right)$

D. $y = 5\cos\left(2x - \frac{\pi}{2}\right)$

---

10. Given $\cot\theta = \frac{1}{3}$. Which of the following is true?

A. $\cos\theta = \pm\frac{\sqrt{10}}{10}$

B. $\sin\theta = \pm\frac{\sqrt{10}}{10}$

C. $\cos\theta = \frac{\sqrt{10}}{10}$

D. $\sin\theta = \frac{3\sqrt{10}}{10}$

Probability Distribution Table

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9031	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9924	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9958	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986

NSW Education Standards Authority

2020 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1}, \quad r \neq 1$$

$$S = \frac{a}{1 - r}, \quad |r| < 1$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

$$\text{and } \alpha\beta\gamma = -\frac{d}{a}$$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

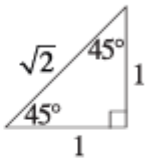
$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

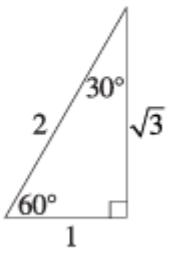
$$A = \frac{1}{2}ab\sin C$$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$



$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$

Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1+t^2}$

$$\cos A = \frac{1 - t^2}{1 + t^2}$$

$$\tan A = \frac{2t}{1 - t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

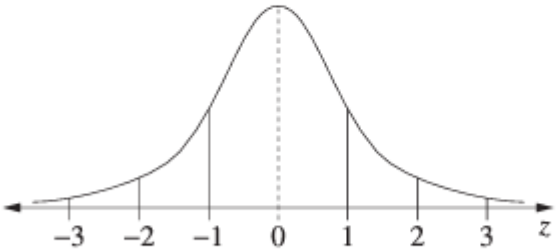
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$$E(X) = \mu$$

$$\text{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$$

Continuous random variables

$$P(X \leq r) = \int_a^r f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r (1 - p)^{n-r}$$

$$X \sim \text{Bin}(n, p)$$

$$\Rightarrow \ P(X = x)$$

$$= \binom{n}{x} p^x (1 - p)^{n-x}, \ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\text{Var}(X) = np(1 - p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\frac{dy}{dx} = nf'(x)[f(x)]^{n-1}$
$y = uv$	$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$y = \frac{u}{v}$	$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\frac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\frac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\frac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\frac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\frac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \cos^{-1} f(x)$	$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \tan^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{1+[f(x)]^2}$

Integral Calculus

$$\int f'(x)[f(x)]^n dx = \frac{1}{n+1}[f(x)]^{n+1} + c \quad \text{where } n \neq -1$$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + c$$

$$\int f'(x)a^{f(x)}dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2-[f(x)]^2}}dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2+[f(x)]^2}dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f(x_1) + \cdots + f(x_{n-1})\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$${}^{n}P_{r} = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^{n}C_{r} = \frac{n!}{r!(n-r)!}$$

$$(x+a)^{n} = x^{n} + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^{r} + \cdots + a^{n}$$

Vectors

$$|\underset{\sim}{u}| = |x\underset{\sim}{i} + y\underset{\sim}{j}| = \sqrt{x^2 + y^2}$$

$$\underset{\sim}{u} \cdot \underset{\sim}{v} = |\underset{\sim}{u}||\underset{\sim}{v}|\cos\theta = x_1x_2 + y_1y_2,$$

where $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$

and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$z = a + ib = r(\cos\theta + i\sin\theta)$$
$$= re^{i\theta}$$

$$\left[r(\cos\theta + i\sin\theta)\right]^{n} = r^{n}(\cos n\theta + i\sin n\theta)$$
$$= r^{n}e^{in\theta}$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$x = a\cos(nt + \alpha) + c$$

$$x = a\sin(nt + \alpha) + c$$

$$\ddot{x} = -n^2(x - c)$$

SECTION II

Student Name: _____

13 marks
Attempt Questions 11 - 16
Allow about 25 minutes on this section.

Teacher Name: _____

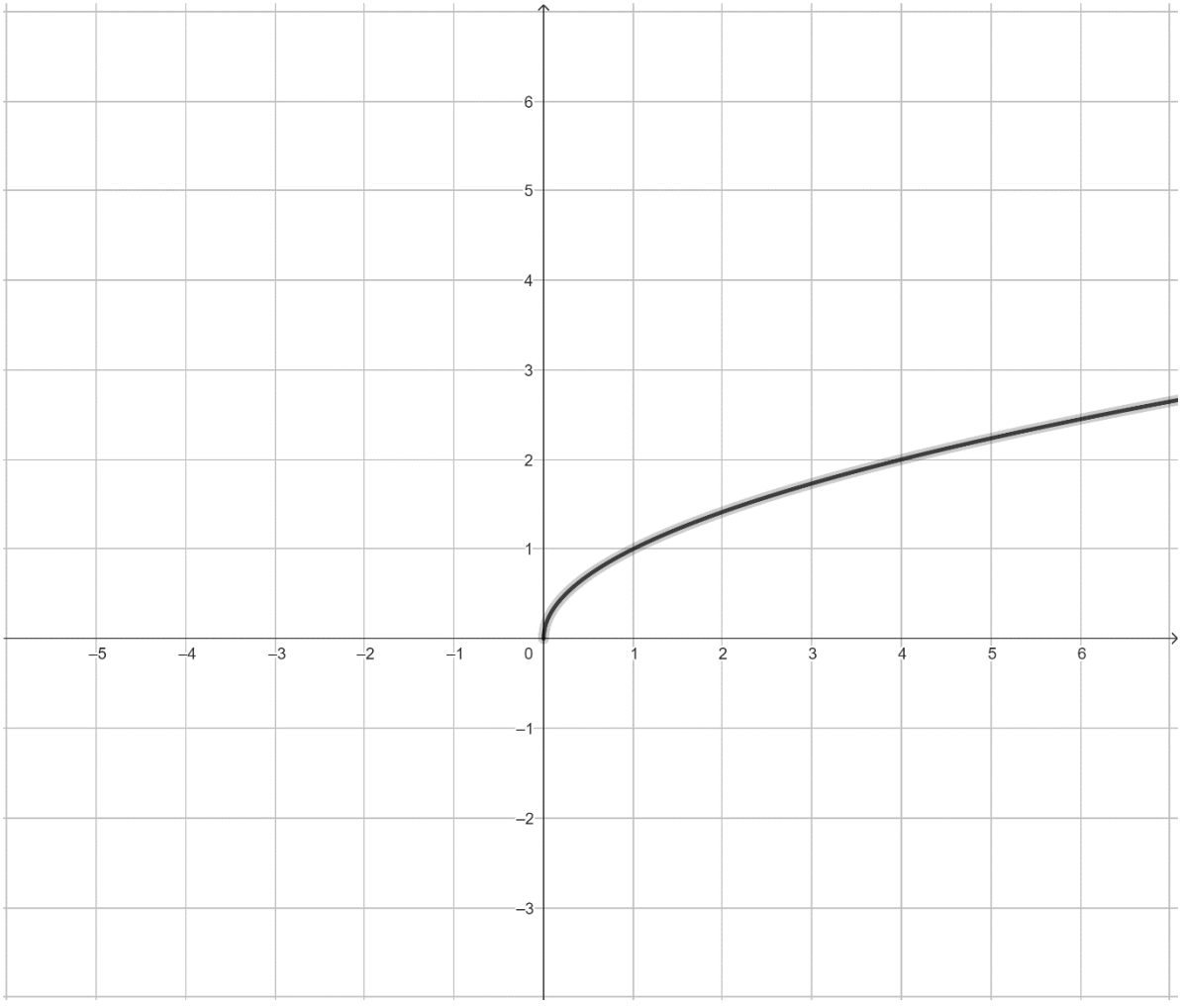
For questions in this section, your responses should include relevant mathematical reasoning and/or calculations.

Answer each question in the spaces provided.

2023 Mathematics Advanced Trial Examination Section II **Marks**

Question 11

The graph of $y = \sqrt{x}$ is shown below.
On the same set of axes, sketch the curve $y = 2\sqrt{x+3} - 1$. **2**



Section II continued on next page ....

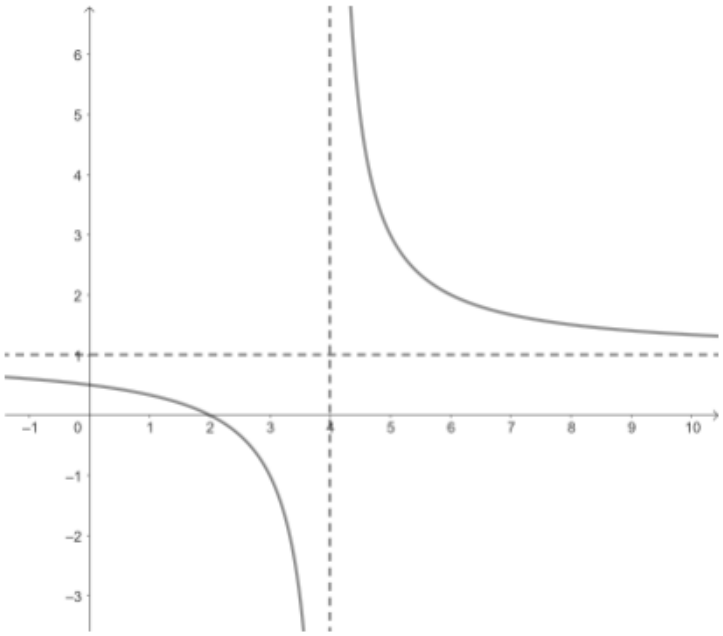
Marks

Question 12

Consider the function $f(x) = \frac{x-2}{x-4}$. The graph of $f(x)$ is shown below.

By using the graph, state the solution to the inequality $\frac{x-2}{x-4} < 3$.

2



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Section II continued on next page ....

Marks

Question 13

Simplify $\ln 60 + \ln e^2 - \ln 15$ **2**

Question 14

Solve the equation $25e^{-0.24t} = 10,$ giving your answer to one decimal place. **2**

Section II continued on next page ....

Marks

Question 15

The wavelength λ in metres of a musical tone is inversely proportional to its frequency f in vibrations per second. The frequency of middle C is approximately 260 vibrations per second and its wavelength is approximately 1.32 m.

a) write an algebraic equation to represent this in the form $\lambda = g(f)$. **1**

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b) Hence find, the frequency, in vibrations per second, of a sound wave with wavelength 0.8 m. **1**

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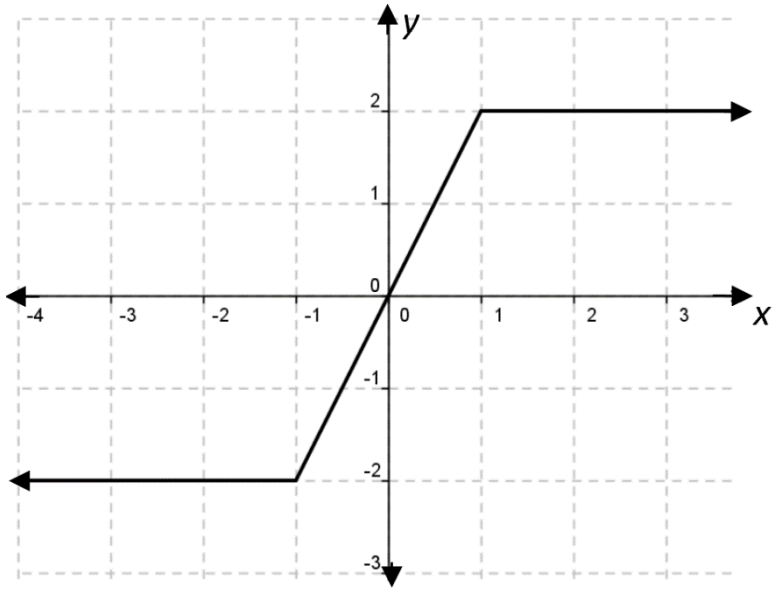
Section II continued on next page ....

Question 16

Consider the graph $y = f(x)$ shown below



a) Evaluate $f(-3) + 2f(2)$ **1**

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b) Determine if the graph drawn is even, odd or neither. Justify your answer. **2**

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SECTION III

Student Name: _____

17 marks
Attempt Questions 17 - 22
Allow about 29 minutes on this section.

Teacher Name: _____

For questions in this section, your responses should include relevant mathematical reasoning and/or calculations.

Answer each question in the spaces provided.

2023 Mathematics Advanced Trial Examination Section III **Marks**

Question 17

Find

$$\int \frac{1}{(5x-7)^3}\,dx$$ **2**

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Section III continued on next page ....

Marks

Question 18

Show that $\frac{x+7}{x-1}$ can be written as $1 + \frac{8}{x-1}$ and hence show that

$$\int_2^{e+1} \frac{x+7}{x-1} dx = e + 7$$

3

Section III continued on next page ....

Marks

Question 19

a) Find the coordinates where the line $y = 2x - 3$ intersects the parabola $y = x^2 - 2x - 3$. **2**

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Question 19 continued on next page ....

Marks

Question 19 continued ....

b) Find the exact area bounded by the line and the parabola. **2**

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Section III continued on next page ....

Marks

Question 20

Evaluate **2**

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\cos x}{\sin x}\, dx$$

Give your answer in exact form.

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Question 21

The gradient function of a curve is given by $\frac{dy}{dx} = 2e^{2x} + 1$. **2**

If the curve passes through the point (0,4), find the equation of the curve.

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Section III continued on next page ....

Question 22

a) Show that $\frac{d}{dx}(\text{cosec } x) = -\cot x \ \text{cosec } x$ **2**

Section III continued on next page ....

Question 22 continued ....

b) Hence, evaluate **2**

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} (\cot x \ \operatorname{cosec} x)\, dx$$

SECTION IV

Student Name: _____

12 marks
Attempt Questions 23 - 25
Allow about 24 minutes on this section.

Teacher Name: _____

For questions in this section, your responses should include relevant mathematical reasoning and/or calculations.

Answer each question in the spaces provided.

2023 Mathematics Advanced Trial Examination Section IV **Marks**

Question 23

A particle is moving along a line. Its displacement (in millimetres) after t seconds is:

$$x(t) = 6t^2 - 1$$

a) Find the average velocity over the interval $[1, 4]$. **1**

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b) Find the instantaneous rate of change of $x(t)$ at $t = 8$. **2**

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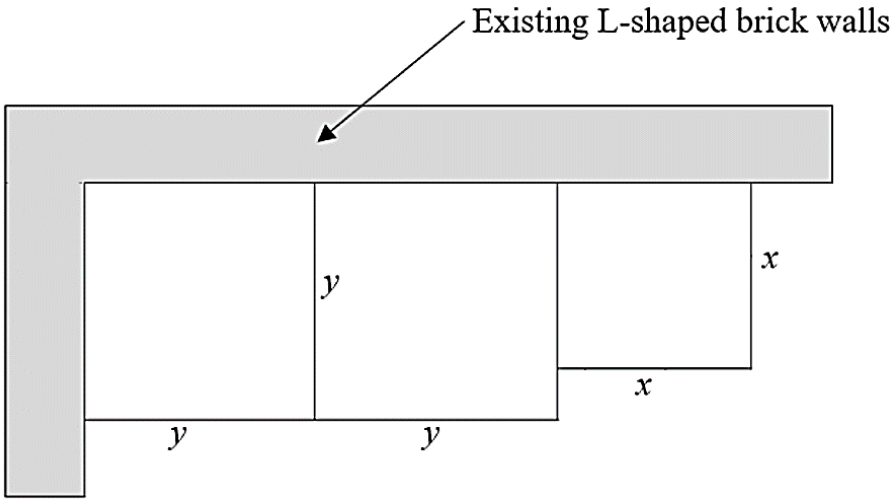
Section IV continued on next page ....

Question 24

The Hurlstone Rural Youth Club wants to make three square enclosures for their new livestock, as shown in the diagram below. They use an existing L-shaped brick wall for some of the sides. They use 36 metres of fencing for all the other sides, as shown, with two of the squares being the same size. Sides x and y are measured in metres.



a) Show that **2**

$$y = \frac{18 - x}{2} .$$

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Question 24 continued on next page ....

Marks

Question 24 continued ...

b) Hence, show that the total area of the enclosure is represented by **2**

$$A = \frac{(18 - x)^2 + 2x^2}{2}.$$

Question 24 continued on next page ....

Marks

Question 24 continued ...

c) Hence, using calculus, calculate the value of x for which the total area of the enclosures is a minimum and find the minimum total area for the three enclosures. **3**

Section IV continued on next page ....

Marks

Question 25

The rate at which water flows into a tank is given by **2**

$$\frac{dV}{dt} = \frac{3t^2}{1 + t^3}$$

where V is the volume of water in the tank in litres and t is the time in seconds.
Initially the tank is empty.

Find V as a function of t.

SECTION V

Student Name: _____

15 marks
Attempt Questions 26 - 29
Allow about 27 minutes on this section.

Teacher Name: _____

For questions in this section, your responses should include relevant mathematical reasoning and/or calculations.

Answer each question in the spaces provided.

2023 Mathematics Advanced Trial Examination Section V **Marks**

Question 26

Differentiate the following functions with respect to x.

a) e^{9-8x} **1**

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b) $log_e(5x^2 + 1)$ **1**

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Section V continued on next page ....

Marks

Question 27

Show that the derivative of $y = x\sqrt{2x+1}$ is given by **2**

$$\frac{dy}{dx} = \frac{3x+1}{\sqrt{2x+1}}.$$

Section V continued on next page ....

Marks

Question 28

For the curve $y = x^3 - 6x^2 + 9x - 4$, $[0, 5]$, find:

a) the coordinates of any stationary points and determine their nature. **3**

Question 28 continued on next page ....

Marks

Question 28 continued ...

b) the coordinates of the point of inflexion. **2**

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c) the absolute maximum of the function. **1**

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Question 28 continued on next page ....

Question 28 continued ...

d) Draw a neat sketch of the curve showing these features. **2**

Section V continued on next page ....

Question 29

Find the equation of the normal to the curve $y = 2\tan 2x$ at the point $\left(\frac{\pi}{2}, 0\right)$. **3**

SECTION VI

Student Name: _____

21 marks
Attempt Questions 30 - 37
Allow about 34 minutes on this section.

Teacher Name: _____

For questions in this section, your responses should include relevant mathematical reasoning and/or calculations.

Answer each question in the spaces provided.

2023 Mathematics Advanced Trial Examination Section VI **Marks**

Question 30

A function is given by

$$f(x) = \begin{cases} \dfrac{3x^2}{125} & 0 \leq x \leq 5 \\ 0 & \text{elsewhere} \end{cases}$$

If this function is a continuous probability distribution, what is the area under the curve? **1**

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Question 31

The weights of the boxes of a certain cereal are normally distributed with a mean of 14 kg and standard deviation of 0.2 kg. The table below shows the weight and z-score of two boxes.

	Weight (kg)	z-score
Box 1	14.15	0.75
Box 2	14.40	1.875

Using the Normal Distribution table find the probability that a randomly selected box of cereal will weigh between 14.15 kg and 14.40 kg. **2**

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Section VI continued on next page ....

Marks

Question 32

Arabi tossed 2 coins to see how many heads he will get. **1**
He knows that this event can be represented by a probability distribution.

Arabi tells his friend Jacob that this is a uniform distribution. Is he correct?
Give a reason for your answer.

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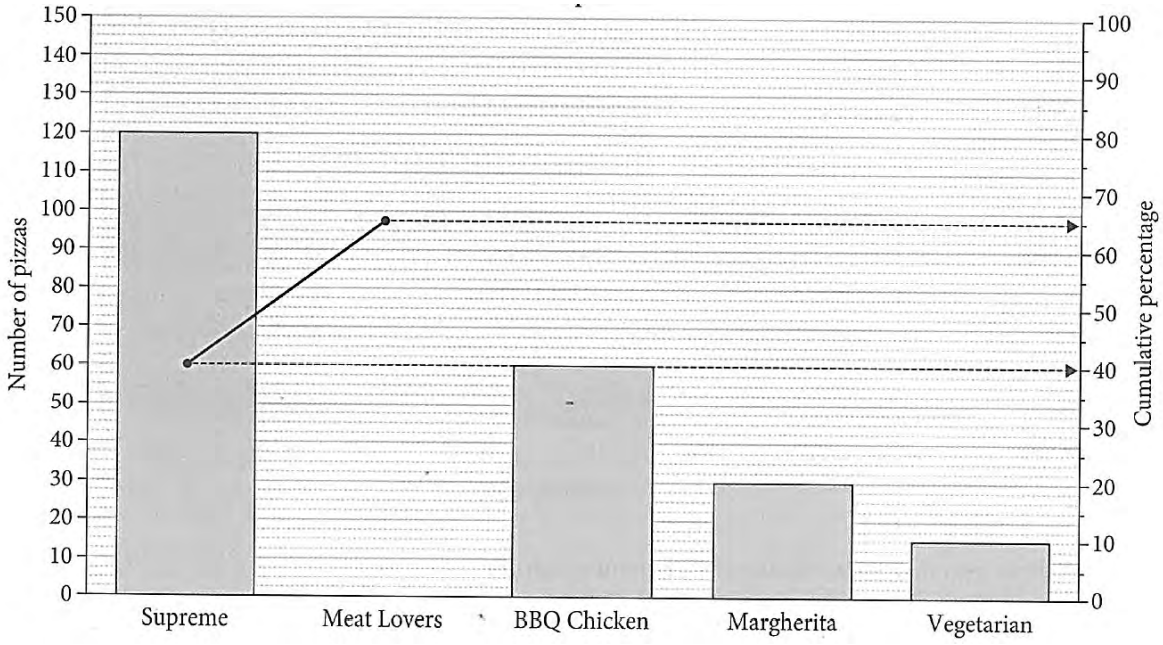
Section VI continued on next page ....

Question 33

Afsheen is a multimillionaire owner of the pizza franchise Feeza Pizza.
Her assistant records the types of pizzas sold in one hour at one of the outlets.
Afsheen draws a Pareto chart displaying the data recorded but leaves it incomplete as she has to go to a meeting.
The Pareto chart she drew is shown below.



a) Complete the chart above by drawing in the column for 'Meat Lovers'. **1**

b) Complete the cumulative frequency line graph on the chart above. **1**

c) Afsheen wants to know the median of the data. Explain why this is not possible. **1**

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Section VI continued on next page ....

Question 34

A continuous random variable X has a cumulative distribution function F(x) given by

$$F(x) = \begin{cases} \frac{1}{3}x^2(4-x^2) & 0 \leq x \leq 1 \\ \\ 0 & \text{otherwise} \end{cases}$$

a) Find $P(X > 0.7)$ **1**

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b) Find the probability density function $f(x)$. **1**

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Question 34 continued on next page ....

Question 34 continued ...

c) Find the mode **2**

Section VI continued on next page ....

Question 35

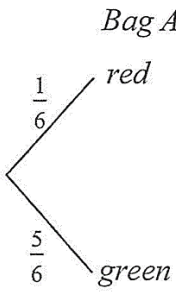
Bag A contains 1 red ball and 5 green balls. Bag B contains 1 red ball and 3 green balls.
Bag C contains 1 red ball and 2 green balls.

A ball is chosen at random from Bag A.

If this ball is red, no further balls are chosen, but if the ball from bag A is green, then a ball is chosen at random from bag B. If this ball is red, no further balls are chosen, but if this ball from bag B is green, then a ball is chosen from bag C.

Let the discrete random variable X denote the number of green balls chosen.

a) Complete the probability tree diagram below. **1**



b) Hence or otherwise draw a probability distribution table for the discrete random variable X. **2**

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Section VI continued on next page ....

Question 36

Elayne recorded the number of push-ups and the number of sit-ups each of her classmates could do in a minute, as seen in the table below.

Push-Ups	Sit-Ups
8	18
10	17
17	22
22	30
29	25
36	47
40	50
48	48
51	57
60	81

a) Elayne spilt coffee on the paper on which she wrote the Pearson Correlation Coefficient. Find the correlation coefficient (r) for Elayne and explain what it means in the context of this data. **2**

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b) Use your calculator to find the equation of the least-squares regression line in the form $y = Bx + A$. Round each value to 1 decimal place. **1**

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Section VI continued on next page ....

Question 37

Ricky sits his trial exams in Gaming in Maths and Mathematical Calligraphy.

The marks for the Mathematical Calligraphy class have a mean of 54 and standard deviation of 5.6.

The marks for the Gaming in Mathematics class have a mean of 76 and standard deviation of 2.1.

a) Compare the distribution of marks for the two classes. **2**

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b) Ricky scored 65 for Mathematical Calligraphy and 80 for Gaming in Mathematics. **2**
Using calculations, explain which subject he performed better in and why.

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SECTION VII

Student Name: _____

12 marks
Attempt Questions 38 - 42
Allow about 26 minutes on this section.

Teacher Name: _____

For questions in this section, your responses should include relevant mathematical reasoning and/or calculations.

Answer each question in the spaces provided.

2023 Mathematics Advanced Trial Examination Section VII **Marks**

Question 38

Prove the following trigonometric identity: **3**

$$\frac{\tan x}{\sec x - 1} - \frac{\sec x - 1}{\tan x} = 2 \cot x$$

Section VII continued on next page ....

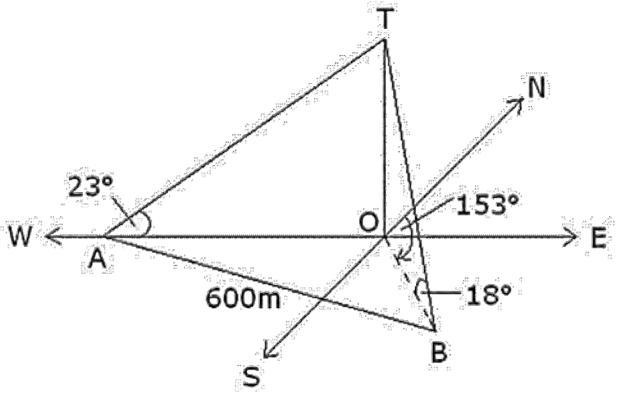
Marks

Question 39

The angle of elevation of the top of a building from a point A due west of the building is 23°. Point B has a bearing of 153° T from the base of the building and the angle of elevation to the top of the building is 18°.

The points A and B are 600 m apart, as shown in the diagram below.

Let $OT = h$.



a) Show that $\angle AOB = 117°$. **1**

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Question 39 continued on next page ....

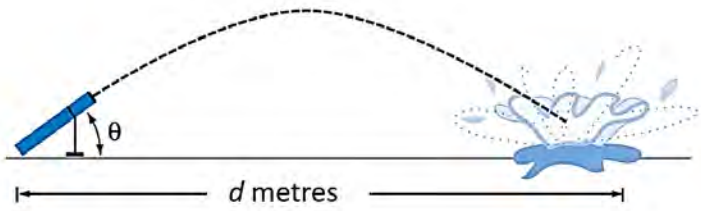
Question 39 continued ...

b) It is given that $AO = h \tan 67^\circ$ and $BO = h \tan 72^\circ$. **3**
Hence, evaluate h to the nearest metre.

Section VII continued on next page ....

Question 40

Anuva and Jaifa were playing with an air cannon. A water balloon leaves the air cannon at an angle of θ with the ground and at an initial velocity of v metres per second. The distance d metres travelled by the water balloon is given by the formula, $d = \frac{1}{32}v^2 \sin 2\theta$.



It is given that the initial velocity of the water balloon is 12 metres per second.
At what angle/s must Anuva adjust the air cannon so that the water balloon lands on Jaifa who is standing 3 metres from the air cannon. **2**

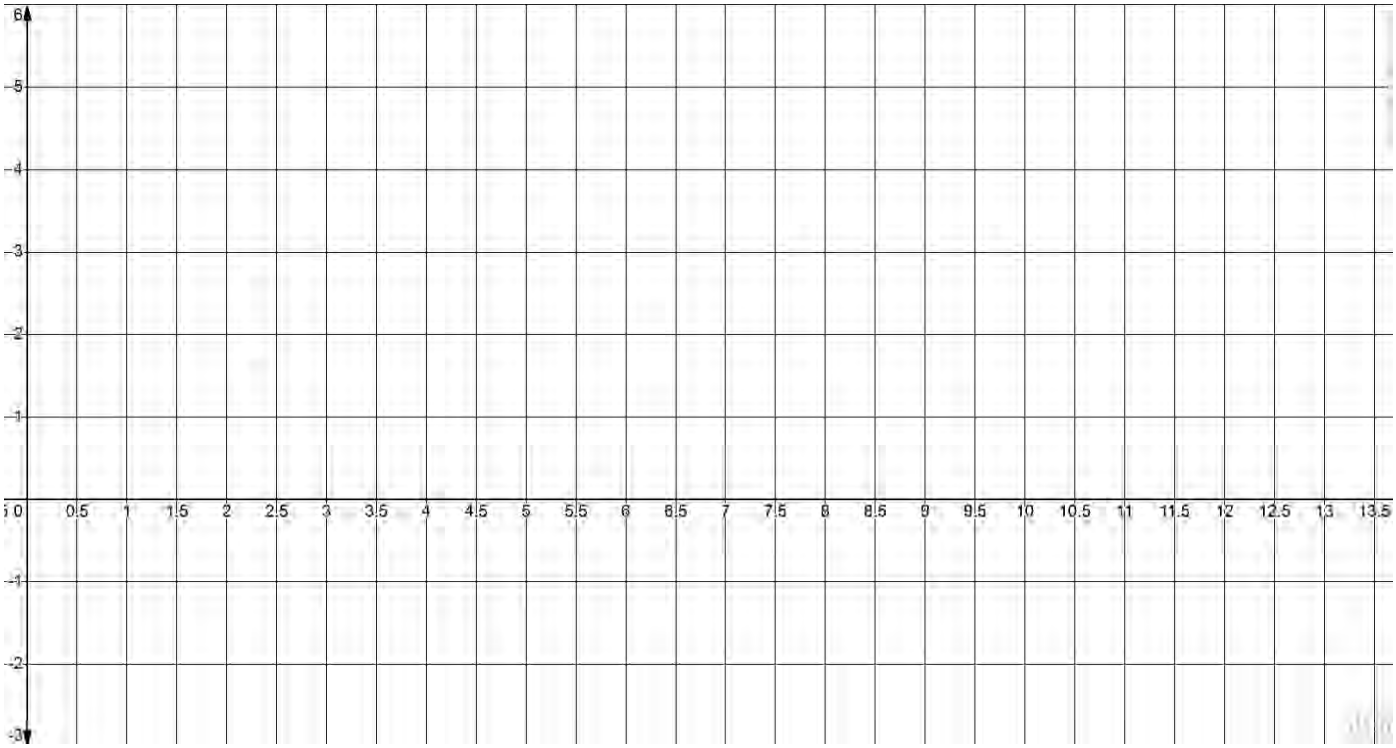
Give your answer to the nearest degree. Assume $0^0 \leq \theta \leq 90^0$.

Section VII continued on next page ....

Question 41

Most stars have at least some variation in luminosity, but unless the fluctuation is large enough to be seen from Earth, the star isn't classified as variable.
The changes in brightness of variable stars aren't generally noticeable to the unaided eye, even if the brightness does change over short timescales. To observe most variable stars, you need to monitor the brightness of the star carefully over extended periods of time.
But there are examples of stars whose brightness has noticeably faded, over short timescales.
One famous example is the red super giant star Betelgeuse.
Its brightness fluctuates over time, following the equation $y = 2 + 3\sin(\frac{\pi x}{3})$ where y is the magnitude of the brightness and x is the number of days.

a) Sketch the graph of $y = 2 + 3\sin(\frac{\pi x}{3})$, for $0 \leq x \leq 4\pi$ on the grid given below. **2**
Show all x and y intercepts.



b) Hence or otherwise calculate how many days it takes for the star to go from its dimmest to brightest and back to dimmest again. **1**

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~ End of Section VII ~

HURLSTONE AGRICULTURAL HIGH SCHOOL

STUDENT'S NAME: _____

TEACHER'S NAME: _____

2023

HSC ASSESSMENT TASK 4
TRIAL EXAMINATION

Mathematics Advanced

Section I - MULTIPLE-CHOICE ANSWER SHEET

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2+4=$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

A ⊗ B ⊗ (correct) C ○ D ○

1.	A ○	B ○	C ○	D ○
2.	A ○	B ○	C ○	D ○
3.	A ○	B ○	C ○	D ○
4.	A ○	B ○	C ○	D ○
5.	A ○	B ○	C ○	D ○
6.	A ○	B ○	C ○	D ○
7.	A ○	B ○	C ○	D ○
8.	A ○	B ○	C ○	D ○
9.	A ○	B ○	C ○	D ○
10.	A ○	B ○	C ○	D ○

Year 12 Mathematics Advanced Trial 2023		
Section I	Multiple Choice Questions ANSWERS	
	Answers	

Multiple Choice:

Q1

Substituting $y = -4$ & solving $-4 = 5 - x, x = 9$.

Substituting $y = 5$ & solving $5 = 5 - x,\ x = 0$, but not including 0 as -4 not included. $\therefore$ domain is $(0, 9]$

Answer: **A**

Q2

The graph of $P(x) = (3 - x)(x + 2)^2$ has a double root at $x = -2$ and a single root at $x = 3$. As the coefficient of x^3 is negative, we begin drawing the graph from the bottom right corner.

Answer: **C**

Q3

Answer: **C**

$$\int \sin\left(\frac{x}{5}\right) dx = -\frac{\cos\left(\frac{x}{5}\right)}{\frac{1}{5}} + C$$

$$= -5\cos\left(\frac{x}{5}\right) + C$$

Q4

Answer: **A**

Q5

Answer: **C**

Q6

Answer: **B**

Q7

Answer: **D**

Q8

Answer: **C**

Q9

Answer: D

Amplitude: 5

Period: For $y = 5cosx$ distance from a maximum value at (0,5) to the next maximum value $(2\pi, 5)$ is 2π units.

For the curve in question distance between maximum and minimum $\frac{3\pi}{4} - \frac{\pi}{4} = \frac{\pi}{2}$

i.e Period in this situation is $= 2 \times \frac{\pi}{2} = \pi \; units$

$\therefore y = 5\cos(2x)$

Phase shift: the maximum value is shifted from (0,5) to $\left(\frac{\pi}{4}, 5\right)$

$$\therefore y = 5\cos\left(2\left(x - \frac{\pi}{4}\right)\right) = 5\cos\left(2x - \frac{\pi}{2}\right)$$

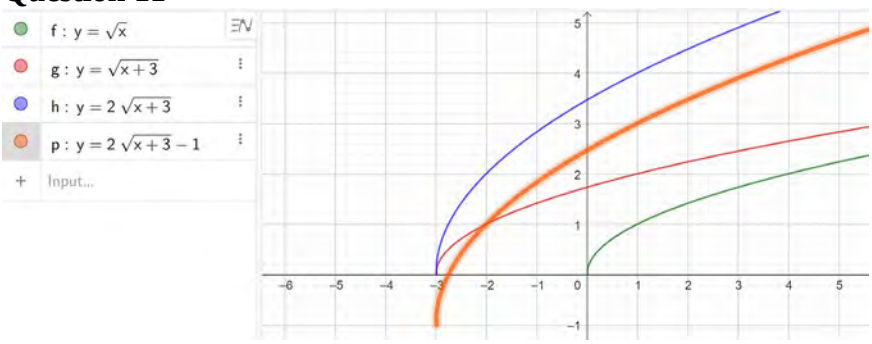
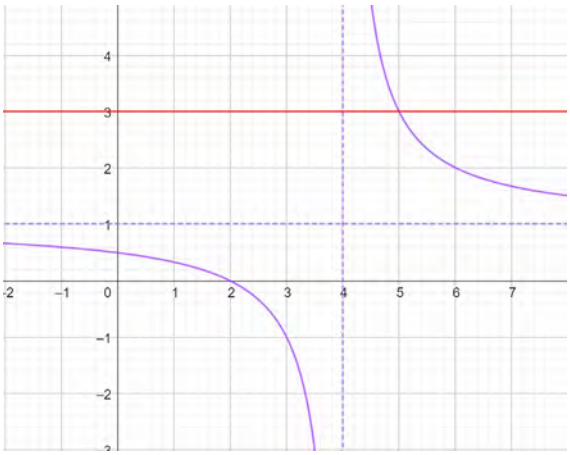
Q10

Answer: A

$cot\theta = \frac{1}{3}$

$\tan\theta = 3$

$\therefore sin\theta = \pm\frac{3}{\sqrt{10}} = \pm\frac{3\sqrt{10}}{10} or\ cos\theta = \pm\frac{1}{\sqrt{10}} = \pm\frac{\sqrt{10}}{10}$

Year 12 Mathematics Advanced Trial 2023		
Section II	Solutions and Marking Guidelines	
Outcomes Addressed in this Question		
MA12-1 Uses detailed algebraic and graphical techniques to critically construct, model and evaluate arguments in a range of familiar and unfamiliar contexts		
Outcome	**Solutions**	**Marking Guidelines**
MA12-1	**Question 11** 	2 marks: correct solution 1 mark: substantial progress towards correct solution
MA12-1	**Question 12** Drawing the line $y = 3$, which meets $y = \frac{x-2}{x-4}$ when $x = 5$, $\frac{x-2}{x-4} = 3$ when $x = 5$.  $\frac{x-2}{x-4} < 3$ when the graph of $y = \frac{x-2}{x-4}$ is below the graph of $y = 3$. This occurs when $x < 4$ or $x > 5$. Note: question said using the graph – if working indicated this was not the case, no marks were awarded.	2 marks: correct solution 1 mark: substantial progress by using the graph towards correct solution
MA12-1	**Question 13** $\ln 60 + \ln e^2 - \ln 15 = \ln 60 - \ln 15 + 2\ln e$ $= \ln \frac{60}{15} + 2 \times 1$ $= \ln 4 + 2$	2 marks: correct solution 1 mark: substantial progress towards correct solution
MA12-1	**Question 14** $25e^{-0.24t} = 10$ $e^{-0.24t} = 0.4$ $\log_e e^{-0.24t} = \log_e 0.4$ $-0.24t = \log_e 0.4$ $t = \frac{\log_e 0.4}{-0.24}$ $t = 3.8$	2 marks: correct solution 1 mark: substantial progress towards correct solution

	Question 15	
MA12-1	a) $\lambda = k \times \frac{1}{f}$ Given $f = 260$ when $\lambda = 1.32$, $1.32 = \frac{k}{260}$, $k = 343.2$ $\therefore\ \lambda = \frac{343.2}{f}$	1 mark: correct answer 1 mark: correct answer or equivalent
	b) If $\lambda = 0.8$, $0.8 = \frac{343.2}{f}$ $0.8f = 343.2$ $\therefore$ frequency is 429 vibrations per second	
	Question 16	
MA12-1	a) $f(-3) + 2f(2) = -2 + 2 \times 2$ $= 2$	
	b) The graph is an odd function, since it has point symmetry about the origin	1 mark: correct answer 2 marks: correct solution 1 mark: identifies function as odd, without sufficient reasoning or correctly describes symmetry with incorrect classification.

Year 12 Mathematics Advanced Assessment Task 4 2023

Section 3 Solutions and Marking Guidelines

Outcomes Addressed in this Question

MA12-7 applies the concepts and techniques of indefinite and definite integrals in the solution of problems

Solutions	Marking Guidelines
Question 17 $\int \frac{1}{(5x-7)^3}dx = \int (5x-7)^{-3}dx$ $= \frac{(5x-7)^{-2}}{-2\times 5} + C$ $= -\frac{1}{10}(5x-7)^{-2} + C$ $= -\frac{1}{10(5x-7)^2} + C$	**2 Marks** Correctly integrates function AND includes constant of integration. **1 Mark** Some progress towards correct solution.
Question 18 $\frac{x+7}{x+1} = \frac{x-1+8}{x-1}$ $= \frac{x-1}{x-1} + \frac{8}{x-1}$ $= 1 + \frac{8}{x-1}$ $\int_2^{e+1} 1 + \frac{8}{x-1}dx$ $= \left[x + 8\ln\lvert x-1\rvert\right]_2^{e+1}$ $= e + 1 + 8\ln\lvert e+1-1\rvert - (2 + 8\ln\lvert 2-1\rvert)$ $= e + 1 + 8\ln e - 2 - 8\ln 1$ $= e + 7$	**3 Marks** Successfully proves the result AND correctly integrates the function AND obtains the correct result. **2 Marks** Substantial progress towards correct solution. **1 Mark** Some progress towards correct solution.

Question 19

$$2x - 3 = x^2 - 2x - 3$$

$$x^2 - 4x = 0$$

$$x(x - 4) = 0$$

$$x = 0, 4$$

When $x = 0, y = 2(0) - 3 = -3$

When $x = 4, y = 2(4) - 3 = 5$

$\therefore$ The points of intersection are $(0, -3)$ and $(4,5)$

2 Marks
Provides correct coordinates of both points of intersection.

1 Mark
Provides only one coordinate.

Question 20

$$\int_0^4 2x - 3 - (x^2 - 2x - 3)dx$$

$$= \int_0^4 4x - x^2 dx$$

$$= \left[2x^2 - \frac{x^3}{3}\right]_0^4$$

$$= 2(4)^2 - \frac{(4)^3}{3} - (2(0)^2 - \frac{(0)^3}{3}$$

$$= \frac{32}{3}\ u^2$$

2 Marks
Successfully creates new integral by subtracting the appropriate functions AND correctly evaluates the exact area (with units)

1 Mark
Some progress towards the correct solution OR missing unit.

Question 21

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos x}{\sin x} = \left[\ln|\sin x|\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$= \ln|\sin\frac{\pi}{2}| - \ln|\sin\frac{\pi}{6}|$$

$$= \ln 1 - \ln\frac{1}{2}$$

$$= -\ln\frac{1}{2}$$

2 Marks
Correctly integrates the function and evaluates the definite integral, leaving solution in exact value.

1 Mark
Some progress towards the correct solution.

$$= -\ln 1 - -\ln 2$$

$$= \ln 2$$

Question 22

2 Marks
Correctly integrate the gradient function with constant of integration AND determines the equation of the curve.

1 Mark
Some progress towards the correct solution OR did not write the equation of the curve.

$$\frac{dy}{dx} = 2e^{2x} + 1$$

$$y = \int 2e^{2x} + 1\, dx$$

$$y = e^{2x} + x + C$$

When $x = 0, y = 4$,

$$4 = e^{2(0)} + 0 + C$$

$$C = 3$$

$$\therefore y = e^{2x} + x + 3$$

Question 23 (a)

2 Marks
Correctly differentiates the function using the chain rule or quotient rule and proves the result.

1 Mark
Some progress towards the correct solution.

$$\frac{d}{dx}(cosec\ x) = \frac{d}{dx}\left(\frac{1}{\sin x}\right) = \frac{d}{dx}(\sin x)^{-1}$$

$$= -1(\sin x)^{-2} \times \cos x$$

$$= -\cos x(\sin x)^{-2}$$

$$= -\frac{\cos x}{sin^2 x}$$

$$= -\frac{\cos x}{\sin x} \times \frac{1}{\sin x}$$

$$= -\cot x\ cosec\ x$$

Question 23 (b)

2 Marks
Correctly uses part (a) to set-up and evaluate the definite integral.

1 Mark
Some progress towards the correct solution.

Using the result from part (a)

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \cot x\ cosec\ x\ dx = \left[-cosec\ x\right]_{\frac{\pi}{6}}^{\frac{\pi}{4}}$$

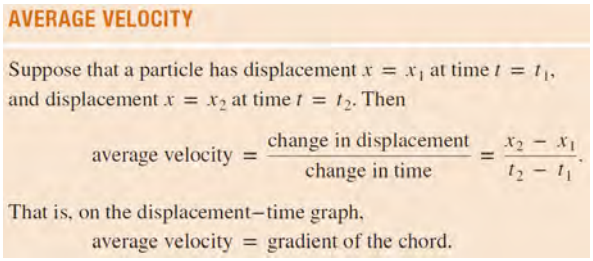
$$= -cosec\ \left(\frac{\pi}{4}\right) - \left(-cosec\ \left(\frac{\pi}{6}\right)\right)$$

$$= -\sqrt{2} + 2 \text{ or equivalent}$$

Year 12 Mathematics Advanced
2023 HSC Assessment Task 4 Trial Examination

Section IV Solutions and Marking Guidelines

Outcome assessed:
MA12-3 applies calculus techniques to model and solve problems

Question	Marking Guidelines
Question 23 a)  **AVERAGE VELOCITY** Suppose that a particle has displacement $x = x_1$ at time $t = t_1$, and displacement $x = x_2$ at time $t = t_2$. Then $$\text{average velocity} = \frac{\text{change in displacement}}{\text{change in time}} = \frac{x_2 - x_1}{t_2 - t_1}.$$ That is, on the displacement–time graph, average velocity = gradient of the chord. $s(1) = 6(1)^2 - 1 = 5$ $s(4) = 6(4)^2 - 1 = 95$ Average Velocity $= \frac{95-5}{4-1}$ $= 30m/s$	1 Mark for using correct definition of average velocity to find the average velocity **Note: Must have correct solution that uses the definition to get 30*m/s*** 2 Marks for complete correct solution 1 Mark for $s'(t) = 12t$
b) $s'(t) = 12t$, at $t = 8$ $s'(8) = 12(8)$ $= 96$ Therefore, the instantaneous rate of change of $s(t)$ at $t = 8$ is 96.	
Question 24 a) Length of fencing: $4y + 2x = 36$ $2y + x = 18$ $2y = 18 - x$ $y = \frac{18-x}{2}$	2 Marks for complete correct solution 1 Mark for $4y + 2x = 36$ or equivalent merit
b) $A = 2y^2 + x^2$ $= 2\left(\frac{18-x}{2}\right)^2 + x^2$ $= \frac{2(18-x)^2}{4} + x^2$ $= \frac{(18-x)^2}{2} + x^2$ $= \frac{(18-x)^2+2x^2}{2}$	2 Marks for complete correct show 1 Mark for $A = 2y^2 + x^2$ or equivalent merit

c) $A = \frac{(18-x)^2+2x^2}{2}$

$= \frac{324-36x+x^2+2x^2}{2}$

$= \frac{324-36x+3x^2}{2}$

$= 162 - 18x + \frac{3}{2}x^2$

$\frac{dA}{dx} = -18 + 2 \times \frac{3x}{2}$

$= 3x - 18$

$3x - 18 = 0$

$3x = 18$

$x = 6\text{m}$

$\frac{d^2A}{dx^2} = 3 > 0 \; so \; minimum \; area$

$A = \frac{(18-6)^2+2\times6^2}{2} = 108\text{m}^2$

3 Marks for complete correct solution

2 Marks for substantial working that could lead to a correct solution with only one error

1 Mark for correctly differentiating the area or equivalent merit

Question 25

$\frac{dV}{dt} = \frac{3t^2}{1+t^3}$

$V = \int \frac{3t^2}{1+t^3}\, dt$

$V = ln|1+t^3| + C$

At $t = 0, V = 0$ $\quad 0 = ln|1+0^3| + C \quad \therefore C = 0$

$\therefore \; V = ln|1+t^3|$

2 Marks for complete correct solution

1 Mark for $V = ln|1+t^3| + C$ or equivalent merit

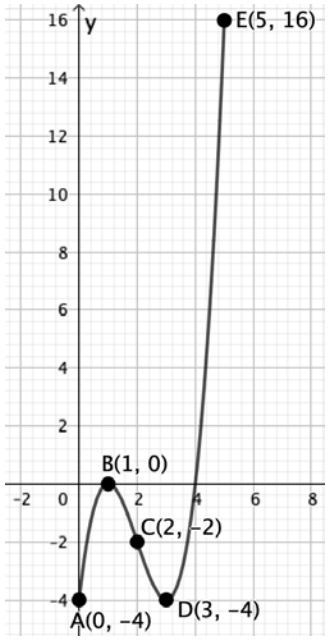
Year 12 Mathematics Assessment Task 4 Trial Examination 2023

Section 5 Questions 26 – 29 Solutions and Marking Guidelines

Outcomes Addressed in this Question

MA12-6 applies appropriate differentiation methods to solve problems

Outcome	Solutions	Marking Guidelines
	26.	**1 mark** Correct answer
MA12-6	**a)** $\frac{d}{dx}e^{9-8x} = -8e^{9-8x}$	
MA12-6	**b)** $\frac{d}{dx}log_e(5x^2+1) = \frac{10x}{(5x^2+1)}$	**1 mark** Correct answer
MA12-6	**27.** $y = x\sqrt{2x+1}$ $= x(2x+1)^{\frac{1}{2}}$ $u = x \qquad v = (2x+1)^{\frac{1}{2}}$ $\frac{du}{dx} = 1 \qquad \frac{dv}{dx} = \frac{1}{2}(2x+1)^{\frac{-1}{2}} \times 2$ $= \frac{1}{\sqrt{2x+1}}$ $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ $= x \times \frac{1}{\sqrt{2x+1}} + (2x+1)^{\frac{1}{2}} \times 1$ $= \frac{x}{\sqrt{2x+1}} + \frac{\sqrt{2x+1}}{1} \times \frac{\sqrt{2x+1}}{\sqrt{2x+1}}$ $= \frac{x+2x+1}{\sqrt{2x+1}}$ $= \frac{3x+1}{\sqrt{2x+1}}$ as required	**2 marks** Correct solution. **1 mark** Substantial progress towards correct solution
	28.	
MA12-6	**a)** $y = x^3 - 6x^2 + 9x - 4$ $y' = 3x^2 - 12x + 9$ $= 3(x^2 - 4x + 3)$ $= 3(x-1)(x-3)$ $y'' = 6x - 12$ Stationary points occur where $y' = 0$ ie. $3(x-1)(x-3) = 0$ $\therefore x = 1, 3$ when $x = 1, y = 0, \quad y'' = 6 - 12$ $= -6 < 0$ $\therefore$ Relative Maximum at $(1, 0)$ when $x = 3, y = -4, \quad y'' = 18 - 12$ $= 6 > 0$ $\therefore$ Relative Minimum at $(3, -4)$	**3 marks** Correct solution stating coordinates of stationary points and correct classification **2 marks** Substantial progress towards correct solution. **1 mark** Some progress towards correct solution.

MA12-6	**b)** Possible inflexions occur when $y'' = 0$ ie. $6x - 12 = 0$ $x = 2,\ y = -2$ x 1 2 3 y'' −6 0 6 sign change in $y'' \rightarrow$ change in concavity $\therefore$ Point of inflexion at $(2, -2)$	**2 marks** Correct solution stating correct coordinates of point of inflexion. **1 mark** Substantial progress towards correct solution. **Note:** No marks awarded here for concavity check but consider including one in this type of question.
MA12-6	**c)** Since domain of function is restricted [0, 5], RH tail of graph may be an absolute maximum ie. when $x = 5, y = 16$ $\therefore$ Maximum value of the function is 16 when $x = 5$	**1 mark** Correct answer. A value, not a point.
MA12-6	**d)**  	**2 marks** Correctly drawn graph showing stationary points, point of inflexion and end points. **1 mark** Substantially correct graph.
MA12-6	**29.** $y = 2tan2x$ $y' = 4sec^2 2x$ Gradient of tangent at $\left(\frac{\pi}{2}, 0\right) = 4sec^2\pi$ $= 4\left(\frac{1}{cos\pi}\right)^2$ $= 4$ Gradient of normal $= -\frac{1}{4}$ Equation of normal $y - y_1 = m(x - x_1)$ $y - 0 = -\frac{1}{4}\left(x - \frac{\pi}{2}\right)$ $-4y = x - \frac{\pi}{2}$ $x + 4y - \frac{\pi}{2} = 0$ OR $y = -\frac{x}{4} + \frac{\pi}{8}$	**3 marks** Correct solution showing all working. **2 marks** Substantial progress towards correct solution. **1 mark** Some progress towards correct solution.

Year 12 Mathematics Advanced 2023 HSC Assessment Task 4 Trial Examination	
Section VI Solutions and Marking Guidelines	
Outcome assessed: **MA12-8** solves problems using appropriate statistical processes	
	Marking Guidelines
Question 30 Area under the curve $= 1$	1 Mark for correct value
Question 31 $$P(0.75 < z < 1.88) = P(z < 1.88) - P(z < 0.75)$$ $$= 0.9699 - 0.7734$$ $$= 0.1965$$	2 Marks for complete correct solution 1 mark for part of the solution correct
Question 32 Arabi is not correct. $P(0\ H)\ +\ P(1\ H)\ +\ P(2\ H)$ are not equal and so cannot be a uniform distribution.	1 Mark for correct statement and reason
Question 33 a) and b)  	1 mark for Correct column drawn for Meat Lovers 1 Mark for cumulative frequency line graphs
c) Since data is categorical, median cannot be found	1 mark for identifying data as categorical and median cannot be found

Question 34

a)

$$P(X < 0.7) = \frac{1}{3}(0.7)^2(4 - (0.7)^2) = 0.5733$$

$$\therefore P(X > 0.7) = 1 - P(X < 0.7) = 0.4267$$

1 Mark for correctly calculating the value of $P(X > 0.7)$

b)

$$F(x) = \frac{4}{3}x^2 - \frac{1}{3}x^4$$

$$f(x) = \frac{8}{3}x - \frac{4}{3}x^3$$

1 Mark for correctly differentiating $F(x)$

c)

$$f(x) = \frac{8}{3}x - \frac{4}{3}x^3$$

$$f'(x) = \frac{8}{3} - 4x^2$$

2 Marks for complete correct solution

1 Mark for some progress towards correct solutions

Mode is when $f'(x) = 0$

$$\frac{8}{3} - 4x^2 = 0$$

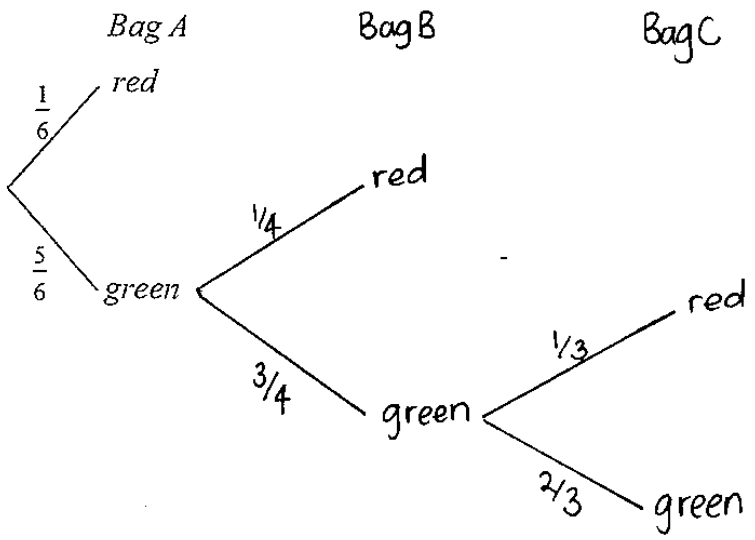
$$\frac{8}{3} = 4x^2$$

$$x = \pm\sqrt{\frac{2}{3}}$$

$$\therefore \text{mode is } x = \sqrt{\frac{2}{3}}$$

Question 35

a)



1 mark for correct probability tree diagram

b)

x	0	1	2	3
$P(x)$	$\frac{1}{6}$	$\frac{5}{24}$	$\frac{5}{24}$	$\frac{5}{12}$

Question 36

a)

$$r = 0.9497$$

There is a strong positive relationship which means people who can do more push-ups can usually do more sit ups.

1 mark for correct value of r

1 mark for explanation

b)

$$y = 1.1x + 4.3$$

1 mark for correct equation

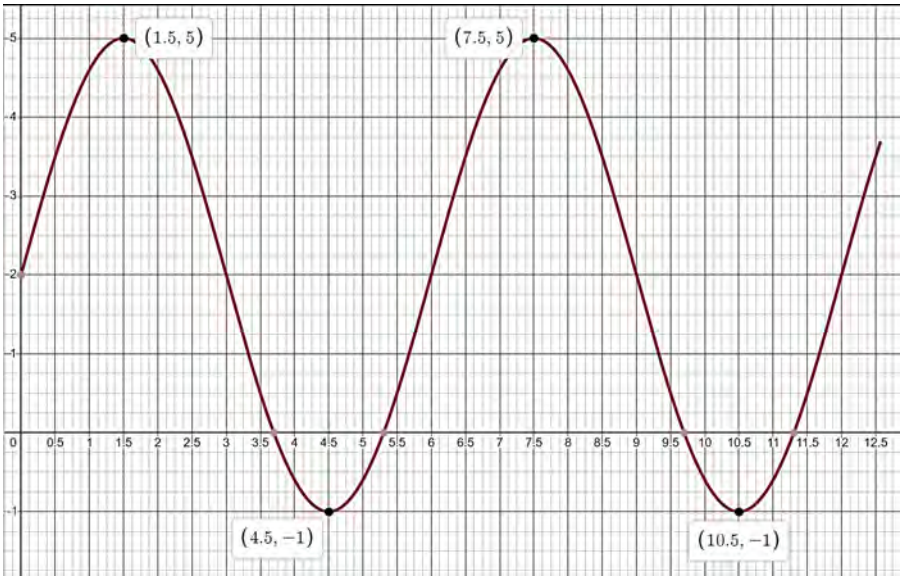
Question 37 a) Gaming has a higher mean and smaller standard deviation meaning scores are grouped closer around the mean. Calligraphy has a lower mean and a larger standard deviation meaning scores are spread out around the mean, less consistent marks.	2 marks for comparing both classes marks mean and standard dev 1 mark for comparing mean or standard deviations for both classes
b) Calligraphy z-score $= \frac{65-54}{5.6} = 1.96$ Gaming z-score $= \frac{80-76}{2.1} = 1.90$ Therefore Ricky performed better in Calligraphy as his z-score was higher.	1 mark for both correct z-scores 1 mark for which class he did better and why

Year 12	Mathematics Advanced 2023	TASK 4
Section VII		

Outcomes Addressed in this Question

MA11-3 :uses the concepts and techniques of trigonometry in the solution of equations and problems involving geometric shapes
MA11-4 :uses the concepts and techniques of periodic functions in the solutions of trigonometric equations or proof of trigonometric identities
MA12-5 :applies the concepts and techniques of periodic functions in the solution of problems involving trigonometric graphs

Part / Outcome	Solutions	Marking Guidelines
Q38 MA11-4	$LHS = \dfrac{\tan x}{\sec x - 1} - \dfrac{\sec x - 1}{\tan x}$ $= \dfrac{\tan^2 x - (\sec x - 1)^2}{\tan x\,(\sec x - 1)}$ $= \dfrac{\tan^2 x - (\sec^2 x - 2\sec x + 1)}{\tan x\,(\sec x - 1)}$ $= \dfrac{\tan^2 x - \sec^2 x + 2\sec x - 1}{\tan x\,(\sec x - 1)}$ $= \dfrac{\tan^2 x - (1 + \tan^2 x) + 2\sec x - 1}{\tan x\,(\sec x - 1)}$ $= \dfrac{2\sec x - 2}{\tan x\,(\sec x - 1)}$ $= \dfrac{2(\sec x - 1)}{\tan x\,(\sec x - 1)}$ $= \dfrac{2}{\tan x} = 2\cot x = \text{RHS}$	**3 marks** – Correct solution **2 marks** – Substantially correct solution. **1 mark** – some correct working towards correct solution
Q39 (a) MA11-3	$\angle AOB = 270° - 153°$ $\therefore \angle AOB = 117°$	**1 mark**– Correct solution
Q39 (b) MA11-3	In ΔAOB, using Cosine Rule, $600^2 = AO^2 + BO^2 - 2(AO)(BO)\cos 117^0$ $600^2 = (htan67^0)^2 + (htan72^0)^2 - 2(htan67^0)(htan72^0)cos117^0$ $600^2 = h^2((tan67^0)^2 + (tan72^0)^2 - 2(tan67^0)(tan72^0)cos117^0)$ $h^2 = \dfrac{600^2}{(tan67^0)^2 + (tan72^0)^2 - 2(tan67^0)(tan72^0)cos117^0}$ $h = \sqrt{\dfrac{600^2}{\tan^2 67^0 + \tan^2 72^0 - 2(tan67^0)(tan72^0)cos117^0}}$ $h \approx 129\ m$ (to the nearest metre)	**3 marks** – Correct solution **2 marks** – Substantially correct solution. **1 mark** – some correct working towards correct solution

Q40 **MA11-4**	Given $d = \frac{1}{32}v^2 sin2\theta$,for $0^0 \leq \theta \leq 90^0$ And $d = 3\ m\ and\ v = 12\ m/s$ $3 = \frac{1}{32} \times 12^2 sin2\theta$ $\sin 2\theta = \frac{2}{3}$ $2\theta = \sin^{-1}\left(\frac{2}{3}\right)$ for $0^0 \leq 2\theta \leq 180^0$ $2\theta = 41.81^0\ or\ 138{,}19^0$ $\theta = 21^0\ or\ 69^0$ to the nearest degree.	**2 marks** – Correct solution **1 mark** – Substantially correct
Q41 (a) **MA12-5**	 	**2 marks** – Correct graph **1 mark** – Substantially correct graph
Q41 (b) **MA12-5**	Hence from graph it takes $10.5 - 4.5 = 6\ days$ Or Alternatively, using the function $y = 2 + 3\sin\left(\frac{\pi x}{3}\right)$, Period can be calculated as $\frac{2\pi}{\frac{\pi}{3}} = 6$. It takes 6 days for the star to go from its dimmest to brightest and back to dimmest again.	**1 mark**– Correct solution